Int. J. Heat Mass Transfer. Vol. 22, pp. 827–830
Pergamon Press Ltd. 1979. Printed in Great Britain

PRESSURE GRADIENTS GENERATED DURING THE DRYING OF POROUS SHAPES

R. D. GIBSON,* M. CROSS† and R. W. YOUNG*

(*Received* 30 *April* 1978 *and in revised form* 9 *August* 1978)

Abstract—A mathematical model is developed to describe the pressure gradients generated during the drying of porous shapes of finite thickness. Simple analytic expressions for the temperature and pressure are derived which allow the important factors influencing the drying process to be highlighted. The model is successfully applied to the drying of dense silica shapes and demonstrates the sensitivity of the pressures generated during drying to the bulk porosity of the shape.

NOMENCLATURE

h, heat-transfer coefficient;
k, permeability;
K, thermal conductivity;
l, shape half-thickness;
L, latent heat;
$p(x,t)$, pressure;
p_A, atmospheric pressure;
R_0, gas constant;
t, time;
$T(x,t)$, temperature;
T_a, ambient temperature;
T_D, drying time;
T_E, evaporation temperature;
$V(x,t)$, velocity;
x, coordinate;
$X(t)$, position of evaporation point.

Greek symbols

$\rho(x,t)$, moisture density;
ρ_w, density of water;
μ, viscosity;
ϕ, porosity.

1. INTRODUCTION

AN IMPORTANT stage in the manufacture of a large number of agglomeration processes is the drying. This is because most "green" agglomerates possess very little natural strength and, if the drying rate is too fast, high pressure gradients may be generated which can give rise to the formation of cracks (or total break-up in the case of iron ore pellets [2]). A comprehensive mathematical description of temperature, moisture and pressure distributions in porous media has been developed by Luikov [3]. Although comprehensive, full utilization of Luikov's system requires the knowledge of a large number of parameters which, in practice, it is not possible to achieve. Recently, however, a simplified approach [4] has been successfully used in calculating the maximum pressure gradients likely to be generated during the drying of iron ore pellets [5].

In this paper, a similar approach is used to model the pressure gradients generated during the drying of a porous shape of finite thickness and infinite height and depth. The analysis is then used with relevant data to assess the pressure gradients likely to be generated during the drying of large dense silica shapes used in coke oven construction.

2. THE MATHEMATICAL MODEL

As the porous refractory shape heats up and dries, the moisture evaporates and the resulting moisture vapour diffuses out towards the surface. The model therefore attempts to describe simultaneously the following aspects of the drying process:

(i) the heat conduction through the shape,
(ii) the evaporation of moisture within the shape and
(iii) the pressure generated by the flow of the moisture vapour through the shape.

To keep the mathematical analysis tractable we only consider one dimensional heat flow and assume that the drying rate is high compared to the rate of moisture diffusion. Figure 1 shows our simplified conception of the drying process (note the symmetry). To minimise the number of parameters that need to be found and to retain only the primary

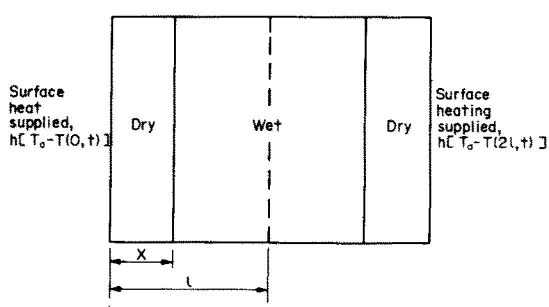

FIG. 1. Schematic diagram showing idealised model of drying process.

*Dept. of Mathematics, Statistics and Computing, Newcastle-upon-Tyne Polytechnic, Ellison Place, Newcastle-upon-Tyne, U.K.
†Dept. of Mathematics and Computer Studies, Sunderland Polytechnic, Priestman Building, Sunderland, U.K.

mechanisms present, the following assumptions are made:

(i) the temperature and moisture distributions are independent of pressure;
(ii) heat flows by conduction only through the shape;
(iii) the shape is divided into two regions containing (a) moisture only at a constant level and (b) moisture vapour only diffusing out;
(iv) the vapour does not take part in the heat transfer process;
(v) the temperature of the vapour is the associated solid temperature;
(vi) the basic interaction between the pressure, temperature and moisture is via the vapour generation at the moving front and
(vii) the movement of the front is slow by comparison with the conduction time.

2.2. *Evaluation of temperature and moisture distributions*

The moving front of moisture gives rise to a moving boundary problem for which Mikhailov and Shishedjiev [6] have developed sophisticated closed form analytic solutions. However under certain conditions a much simpler analytic solution may be derived as follows. Heat is supplied to a shape of semi-width, l and porosity, ϕ, so that

$$K\frac{\partial T}{\partial x}(0,t)+h[T_a-T(0,t)]=0, \tag{1}$$

where K is the thermal conductivity of the brick, h is the heat-transfer coefficient, $T(x,t)$ is the temperature T_a is the outside temperature and since the front $x = X(t)$ is at the evaporation temperature T_E then

$$T[X(t),t]=T_E. \tag{2}$$

For our application, on a thermal time scale, the movement of the front is slow by comparison with the rate of conduction, so that to a first approximation we may assume

$$\frac{\partial^2 T}{\partial x^2}=0, \tag{3}$$

within the dry region [i.e. $0 < x < X(t)$]. Thus, in the dry part of the shape it follows that

$$T(x,t)=T_E+\frac{h(T_a-T_E)}{(K+hX)}(X-x). \tag{4}$$

Further, since the heat supply to the evaporation front is dominant, the speed of the front may be determined from the following heat balance,

$$\phi\rho wL\frac{\mathrm{d}X}{\mathrm{d}t}=-K\frac{\partial T}{\partial x}(X-,t)$$
$$=\frac{Kh(T_a-T_E)}{(K+hX)}, \tag{5}$$

where ρw is the density of water and L is the latent heat of evaporation. The position of the moving front as a function of time is given by integration of equation (5), i.e.

$$\phi\rho wL(KX+\tfrac{1}{2}hX^2)=Kh(T_a-T_E)t, \tag{6}$$

where it is assumed $X(0)=0$. It follows that the drying time, T_D, is given by

$$T_D=\frac{\phi\rho wLl(K+\tfrac{1}{2}hl)}{Kh(T_a-T_E)}. \tag{7}$$

2.3. *Evaluation of the maximum pressure gradients*

The gas flow in the dry part of the shape $[0 < x < X(t)]$ is governed by Darcy's law,

$$V=-\frac{k}{\mu}\frac{\partial p}{\partial x}, \tag{8}$$

where $V(x,t)$ is the speed of the water vapour, $p(x,t)$ is the pressure, k is the permeability and μ the viscosity, together with the continuity equation

$$\frac{\partial}{\partial x}(\rho V)=-\phi\frac{\partial \rho}{\partial t} \tag{9}$$

and the perfect gas law

$$p=R_0\rho T, \tag{10}$$

where R_0 is the gas constant and ρ the vapour density.

For a number of applications [5] the right hand side of equation (9) is relatively small, so that ρV is a function of time only. However, the continuity condition at the moving front implies

$$\rho V=-\rho w\dot{X}, \tag{11}$$

so that using equations (8) and (10) we have

$$p\frac{\partial p}{\partial x}=\frac{R_0\mu\rho w}{k}\dot{X}T. \tag{12}$$

Assuming, that the external pressure p_A is atmospheric, integration of equation (12) yields

$$p^2=p_A{}^2+\frac{2R_0\mu Kh(T_a-T_E)}{k\phi L(K+hX)}\times\left\{xT_E+\frac{h(T_a-T_E)}{(K+hX)}\cdot(X-\tfrac{1}{2}x)x\right\}. \tag{13}$$

The pressure gradient can be obtained from equations (5), (12) and (13).

3. RESULTS AND DISCUSSION

From equation (13) it is readily seen that the maximum pressure gradient will occur right at the surface of the shape. The importance of this observation is that throughout the drying process there will be continuously high body forces near the surface implying that any cracks or lineations formed to relieve this imposed "tensile stress" will not be generated very far beneath the surface.

It is well known that the bulk density (or alternatively porosity) of refractory shapes is a strongly limiting factor in the optimisation of drying times. Figure 2 shows a plot of permeability/viscosity against porosity for standard dense silica shapes.

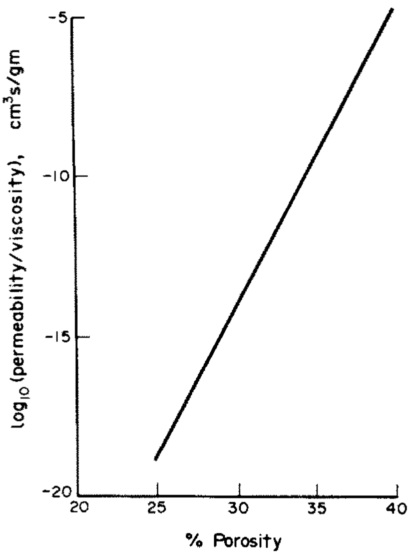

FIG. 2. Permeability/viscosity ratio of a green silica shape as a function of porosity.

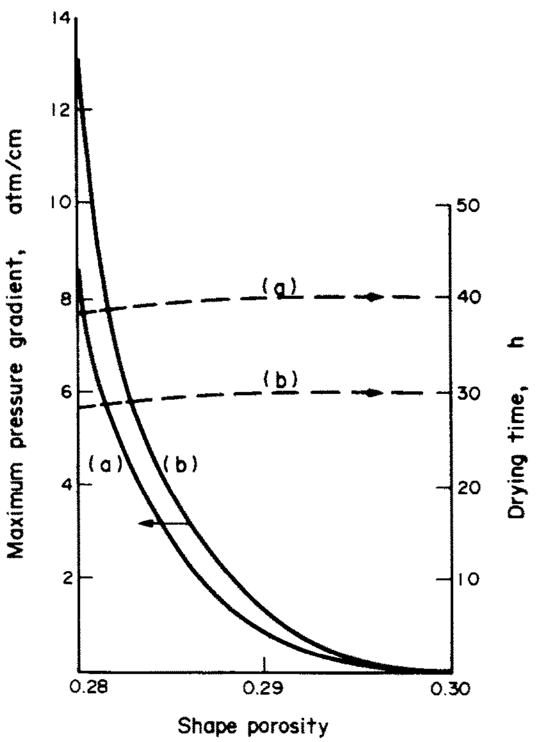

FIG. 3. Variation of maximum pressure gradients and drying times for porosity for two ambient drying temperatures: (a) 110°C; (b) 115°C.

From this plot it is clear that the permeability is very sensitive to the shape porosity. This sensitivity results from the small mean particle size of the raw silica material used in the manufacturing process. This sensitivity is reflected in the evaluation of pressure gradients and, in fact, renders the bulk density of the shape the most significant factor in determining the maximum pressure gradients. Figure 3 illustrates some typical model results for the data shown in Table 1. From this plot and our other work it is clear that the factors affecting the drying time are of minimal importance with respect to the generated pressures when compared to the bulk density. In fact, from Fig. 3 it could be concluded that the pressure gradient would disappear for porosities greater than 0.3. This compares well with the experience of manufacturers of dense silica shapes where it is found that as the porosity increases above 0.3 very little spalling occurs during drying [7].

Table 1. Value of parameters used in calculations

Parameter	Value
h	$0.001\,g\,s^{-3}\,°C^{-1}$
K	$0.004\,g\,cm\,s^{-3}\,°C^{-1}$
l	8.5 cm
L	$530\,cal\,g^{-1}$
T_E	100°C
ρw	$1\,g\,cm^{-3}$

Assuming high pressure gradients can be associated with higher incidences of surface cracking, these results have two main implications for the efficient production of dense silica shapes:

(i) it is vitally important to ensure that correct bulk densities are maintained consistently and

(ii) slower drying rates will not significantly reduce the maximum pressure gradients generated (i.e. intensity of surface cracking).

Acknowledgement—The authors wish to thank J. K. Forster of the British Steel Corporation Teesside Laboratories for providing the relevant data on dense silica shapes.

REFERENCES

1. K. V. S. Sastry (Ed.), *Agglomeration* 77, AIME (1977).
2. N. F. Schulz, H. A. Lex and J. D. Zetterstrom, Decrepitation of balls during pelletization of iron ore, *Trans. SME—AIME* **235**, 251 (1966).
3. A. V. Luikov, Systems of differential equations of heat and mass transfer in capillary—porous bodies, *Int. J. Heat Mass Transfer* **18**, 1 (1975).
4. M. Cross, R. D. Gibson and R. W. Young, Pressure generation during the drying of a porous half-space, *Int. J. Heat Mass Transfer* **22**, 47 (1979).
5. R. D. Gibson, M. Cross and R. W. Young, Pressure generation during the drying of iron ore pellets, *Proc. Int. Conf. Appl. Numerical Modelling* 539, Pentech Press (1978).
6. M. O. Mikhailov and N. K. Shishedjiev, Temperature and moisture distributions during contact drying of a porous sheet, *Int. J. Heat Mass Transfer* **18**, 15 (1975).
7. J. K. Forster, Unpublished.

GRADIENTS DE PRESSION ASSOCIES AU SECHAGE DES CORPS POREUX

Résumé—On développe un modèle mathématique pour décrire les gradients de pression lors du séchage des corps poreux d'épaisseur finie. Des expressions analytiques simples pour la température et la pression sont obtenues et elles dégagent les facteurs importants qui influencent le mécanisme du séchage. Le modèle est appliqué avec succès au séchage des corps denses siliceux et il montre la sensibilité à la pression créée lors du séchage de la masse poreuse.

DRUCKGRADIENTEN IN PORÖSEN KÖRPERN BEIM TROCKNEN

Zusammenfassung—Es wurde ein mathematisches Modell zur Beschreibung der Druckgradienten entwickelt, die beim Trocknen von porösen Körpern endlicher Dicke auftreten. Es wurden einfache analytische Ausdrücke für Temperatur und Druck abgeleitet, welche erlauben, die wesentlichen Parameter hervorzuheben, die den Trockenprozeß beeinflussen. Das Modell wurde erfolgreich auf den Trockenvorgang von Körpern aus dichter Kieselerde angewendet. Es zeigt die Abhängigkeit der Druckverteilung während des Trocknungsvorgangs von der Porosität im Kern des Körpers.

ВОЗНИКНОВЕНИЕ ГРАДИЕНТОВ ДАВЛЕНИЯ ПРИ СУШКЕ ПОРИСТЫХ ТЕЛ

Аннотация — Предложена математическая модель для определения градиентов давления, возникающих при сушке пористых тел конечных размеров. Выведены простые аналитические выражения для определения температуры и давления, позволяющие выделить важные факторы, контролирующие процесс сушки. Модель успешно применена для расчёта сушки плотных кремниевых образцов, при этом показано влияние объёмной пористости образца на градиенты давления, возникающие в процессе сушки.